

MODIFIED FORMALISM FOR THE DESCRIPTION OF MULTIPLE-PULSE
NMR EXPERIMENTS FOR SYSTEMS WITH MAGNETIC EQUIVALENCE

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During the last decade, several density operator formalisms^[1] have been developed for the description of multiple-pulse NMR experiments with more physical transparency and easy analysis. Unfortunately, their applications to spin systems with moderate complexity are still cumbersome.

Here we propose a modified formalism for weakly coupled spin 1/2 systems with magnetic equivalence.

Symmetry Adapted Product Operator (SAPO)^[2]. For I_n spin system, SAPO is constructed by a permutation symmetric combination of the conventional Cartesian product operators as formula (1), which constitute an irreducible base operator of permutation group S_n .

$$\begin{aligned} I^{(n)} &= (1/2)E_n, \quad I_{ii}^{(n)} = \sum_i I_{i_i}, \quad I_{i_ii}^{(n)} = \sum_{i,j} 2I_{i_i} I_{j_j}, \\ I_{i_ii}^{(n)} &= \sum_{i,j} 2(I_{i_i} I_{j_j} + I_{i_j} I_{j_i}), \quad I_{i_ii}^{(n)} = \sum_{i,j,k} 4I_{i_i} I_{j_j} I_{k_k}, \\ I_{i_ii}^{(n)} &= \sum_{i,j,k} 4(I_{i_i} I_{j_j} I_{k_k} + I_{i_i} I_{j_k} I_{k_j} + I_{i_j} I_{j_i} I_{k_k}), \\ I_{i_ii}^{(n)} &= \sum_{i,j,k} 4(I_{i_i} I_{j_j} I_{k_k} + I_{i_j} I_{j_i} I_{k_k} + I_{i_w} I_{j_j} I_{k_k} + \\ &\quad I_{i_i} I_{j_w} I_{k_k} + I_{i_j} I_{j_w} I_{k_k} + I_{i_w} I_{j_j} I_{k_k}) \\ &\quad \text{etc.} \end{aligned} \quad (1)$$

The main advantage of SAPO is markedly reducing the number of base operators, for instance, 16 to 10 for AX, A_2 and 64 to 20 for AMX, A_3 systems respectively. In addition, general cyclic commutation relations exist

among SAPO, which govern the evolution rule of multiple quantum coherences with different orders. SAPO were used to analyse DEPT etc experiments on $A_n X$ systems^[2].

Multiple Quantum Coherence Product Operators (MQCPO). We introduced MQCPO as Hermitian combinations of irreducible tensor operators much like Weitekamp^[3], but with the composite particle spin operators instead of single spin operator:

$$\begin{aligned} K_{X\pm}^{ij} &= I_X^{ij} \pm I_X^{i\bar{j}}, & K_{Y\pm}^{ij} &= I_Y^{ij} \pm I_Y^{i\bar{j}}, \\ K_{Z\pm}^{ij} &= I_Z^{ij} \pm I_Z^{i\bar{j}}, & K_{O\pm}^{ij} &= I_O^{ij} \pm I_O^{i\bar{j}}. \end{aligned} \quad (2)$$

Evolutions of the operator under the unperturbed weak coupling Hamiltonian are trivial (cf. Table 1), especially evolution under $I_z^{(n)}$ is simple. Furthermore it is turned out that there exist simple linear relationships between SAPO and MQCPO. Taking account of the above facts, an action of rf pulse may be treated by representation transformation to Z-representation via cyclic exchange of coordinate indexes of SAPO. Concerned using SAPO and MQCPO made the description of coherence transfer and evolution terse and easy in multiple-pulse experiments for systems with magnetic equivalence. The power of this formalism is demonstrated by analysing spin pattern filtering and heteronuclear spectral editing.

Table 1. Cyclic commutation properties of MQCPO for $A_n X(I_n S)$ systems

$$\begin{aligned} [I_z^{(n)}, K_{X\pm}^{ij}] &= ip_{ij} K_{Y\pm}^{ij}; & [2I_z^{(n)} S_z, K_{X\pm}^{ij}] &= ip_{ij} 2K_{Y\pm}^{ijs} S_z, & [2I_z^{(n)} S_z, 2K_{Z\pm}^{ijs} S_x] &= iq_{ij} 2K_{Z\pm}^{ijs} S_y, \\ [2I_z^{(n)} S_z, 2K_{X\pm}^{ijs} S_z] &= ip_{ij} K_{Y\pm}^{ij}, & [2I_z^{(n)} S_z, 2K_{X\pm}^{ijs} S_x] &= iq_{ij} 2K_{X\pm}^{ijs} S_y, & [2I_z^{(n)} S_z, 2K_{Y\pm}^{ijs} S_x] &= iq_{ij} 2K_{Y\pm}^{ijs} S_y. \end{aligned}$$

$$p_{ij} = m_i - m_j, \quad q_{ij} = m_i + m_j; \quad p_{ij} + q_{ij} > 0$$

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